SET-2

Series BVM/2

Code No. **65/2/2**

Roll No. | | | | | | | |

Candidates must write the Code on the title page of the answer-book

MATHS (Theory) & SOLUTION

Time allowed : 3 hours **Maximum Marks : 100**

General Instructions :

(i) ***All** questions are **compulsory.***

(ii) *The question paper consists of 29 questions divided into four sections A,B,C and D. Section A comprises of 4 questions of 1 mark each, Section B comprises of 8 questio each, Section C comprises of 11 questions of 4 marks each, and Section D comprises 6 questions of 6 marks each.*

(iii) *All questions in Section A are to be answered in one word, one sentence or as requirement of the question.*

(iv) *There is no overall choice. However internal choice has been provided in 1 question of questions of Section B, 3 questions of Section C and 3 questions of Section D. You only one of the alternatives in all such questions.*

(v) *Use of calculators is not permitted. You may ask for logarithmic tables, if required.*

SECTION A

Question number 1 to 4 carry 1 mark each.

Q.1. Find |AB|, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

Ans.

$$AB = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

Q.2. Differentiate $e^{\sqrt{3x}}$, with respect to x.

Ans.

$$y = e^{\sqrt{3x}}$$

$$\frac{dy}{dx} = e^{\sqrt{3x}} \times \frac{1}{2\sqrt{3x}} \times 3$$

$$\frac{dy}{dx} = \frac{3y}{2\sqrt{3x}}$$

Q.3. Find the order and degree (if defined) of the differential equation

$$\frac{d^2y}{dx^2} + y\left(\frac{dy}{dx}\right)^2 = 2x^2 \log\left(\frac{d^2y}{dx^2}\right)$$

Ans. Degree = Not defined, order = 2

Q.4. Find the direction cosines of a line which makes equal angles with the coordinate axes.

OR

A line passes through the point with position vector $2\hat{i}-\hat{j}+4\hat{k}$ and is in the direction of the vector $\hat{i}+\hat{j}-2\hat{k}$. Find the equation of the line in cartesian form.

Ans. $\alpha=\beta=\gamma$

$\ell=m=n$

We know that

$\ell^2+m^2+n^2=1$

$3\ell^2=1$

$\ell=\pm\frac{1}{\sqrt{3}}$

$\ell=m=n=\pm\frac{1}{\sqrt{3}}$

D.C's $\left(\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}}\right)$

OR

Ans. $\vec{a}=2\hat{i}-\hat{j}+4\hat{k}$

$\vec{b}=\hat{i}+\hat{j}-2\hat{k}$

$\vec{r}=\vec{a}+\lambda\vec{b}$

$=2\hat{i}-\hat{j}+4\hat{k}+\lambda\left(\hat{i}+\hat{j}-2\hat{k}\right)$

Eq. of line in cartesian form $\frac{x-2}{1}=\frac{y+1}{1}=\frac{z-4}{-2}$

SECTION B

Question numbers 5 to 12 carry 2 marks each.

Q.5. Find : $\int\sqrt{3-2x-x^2}\,dx$

Ans. $\int\sqrt{3-2x-x^2}\,dx$

$3-2x-x^2=-(x^2+2x-3)$

$=-(x^2+2x+1-4)$

$=-((x+1)^2-2^2)$

$=2^2-(x+1)^2$

$\int\sqrt{3-2x-x^2}\,dx = \int\sqrt{2^2-(x+1)^2}\,dx$

$=\int\sqrt{2^2-(x+1)^2}\,dx$

$=\frac{x+1}{2}\sqrt{3-2x-x^2}+\frac{2^2}{2}\sin^{-1}\frac{(x+1)}{2}+c$

$=\frac{x+1}{2}\sqrt{3-2x-x^2}+2\sin^{-1}\frac{(x+1)}{2}+c$

Q.6. If A = $\begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$ and $|A^3|$ = 125, then find the values of p.

Ans. $A^2 = \begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}\begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix} = \begin{bmatrix} p^2+4 & 4p \\ 4p & 4+p^2 \end{bmatrix}$

$$A^3 = \begin{bmatrix} p^2+4 & 4p \\ 4p & 4+p^2 \end{bmatrix}\begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$$

$$= \begin{bmatrix} p^3+4p+8p & 2p^2+8+4p^2 \\ 4p^2+8+2p^2 & 8p+4p+p^3 \end{bmatrix}$$

$$|A^3| = (p^3 + 12p)^2 - (6p^2 + 8)^2$$

$$|A^3| = (p^3+12p-6p^2-8)(p^3+12p+6p^2+8)$$

$$= (p^3-6p^2+12p-8)(p^3+6p^2+12+8)$$

$$= (p-2)^3(p+2)^3$$

$$= (p^2-4)^3$$

$$(p^2-4)^3 = 125$$

$$p^2-4 = 5$$

$$p^2 = 9$$

$$p = \pm 3$$

Shortcut

$$|A^n| = |A|^n$$

$$|A^3| = |A|^3 = 125$$

$$|A| = 5$$

$$p^2-4 = 5$$

$$p^2 = 9$$

$$p = \pm 3$$

Q.7. Examine whether the operation $*$ defined on R, the set of all real numbers, by $a*b = \sqrt{a^2+b^2}$ is a binary operation or not, and if it is a binary operation, find whether it is associative or not.

Ans. $a \in R, b \in R,$

$$a^2+b^2 \in R$$

$$\sqrt{a^2+b^2} \in R$$

is a binary operation.

$$a*b = \sqrt{a^2+b^2}$$

$$(a*b)*c = \sqrt{a^2+b^2}*c$$

$$= \sqrt{a^2+b^2+c^2}$$

$$a*(b*c) = a*\sqrt{b^2+c^2}$$

$$= \sqrt{a^2+b^2+c^2}$$

$a * (b*c) = (a*b)*c.$

It is associative

Q.8. If $|\vec{a}| = 2,\ |\vec{b}| = 7$ and $\vec{a}\times\vec{b} = 3\hat{i}+2\hat{j}+6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

OR

Find the volume of a cuboid whose edges are given by $-3\hat{i}+7\hat{j}+5\hat{k}$, $-5\hat{i}+7\hat{j}-3\hat{k}$ and $7\hat{i}-5\hat{j}-3\hat{k}$.

Ans. $|\vec{a}| = 2,\ |\vec{b}| = 7$

$\vec{a}\times\vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ $\qquad \sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

$$= \frac{\sqrt{9+4+36}}{2 \times 7}$$

$$= \frac{7}{2\times 7} = \frac{1}{2}$$

$$\theta = \frac{\pi}{6} \text{ or } 30°$$

OR

Ans. $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}$

$\vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}$

$\vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$

Area of cuboid $= \vec{a}.\left(\vec{b}\times\vec{c}\right) = \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix} = -3\,(-21-15) - 7(15+21) + 5(25-49)$

$= -3(-36) - 7(36) + 5(-24)$

$= 108 - 252 - 120$

$= -264$

Since volume cannot be negative the so volume of cuboid is 264 cube unit

Q.9. A coin is tossed 5 times. What is the probability of getting (i) 3 heads, (ii) at most 3 heads?

OR

Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.

Ans. $p = \dfrac{1}{2}$ $\qquad q = \dfrac{1}{2}$ $\qquad$ X = no. of heads

n = 5

(i) $p(x = 3) = {}^5c_3\ p^3q^2$

$$= \frac{5!}{3!2!}\left(\frac{1}{2}\right)^5 = \frac{120}{6\times 2}\times\frac{1}{32} = \frac{10}{32} = \frac{5}{16}$$

(ii) $p\left(x \le 3\right) = 1 - p\left(x = 4\right) - p\left(x = 5\right)$

$1 - {}^5c_4p^4q - {}^5c_5p^5q^0$

$$= 1 - \frac{5!}{4!1!}\left(\frac{1}{2}\right)^5 - \frac{5!}{5!0!}\left(\frac{1}{2}\right)^5$$

$$= 1 - \frac{5}{32} - \frac{1}{32}$$

$$= 1 - \frac{6}{32} = 1 - \frac{3}{16}$$

$$= \frac{13}{16}$$

OR

Ans. X = no. of heads   S = {HH, HT, TH, TT}

n = 2 $p = \frac{1}{2}, \quad q = \frac{1}{2}$

X	P_i
0	1/4
1	2/4
2	1/4

Q.10. Find :

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$$

OR

Find :

$$\int \frac{x-3}{(x-1)^3} e^x \, dx$$

Ans. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \frac{\sin}{\cos^2 x} + \frac{\cos}{\sin^2 x} dx$

$\int \tan x \sec x + \cot x \operatorname{cosec} x \, dx$

= secx – cosecx + c

OR

Ans. $\int e^x \left(\frac{x-3}{(x-1)^3} \right) dx$

$\int e^x \left(\frac{x-1}{(x-1)^3} - \frac{2}{(x-1)^3} \right) dx$

$\int e^x \left(\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right) dx$

$\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$

$= \frac{e^x}{(x-1)^2} + c$

Q.11. If P(not A) = 0.7, P(B) = 0.7 and P(B/A) = 0.5, then find P(A/B).

Ans. P(not A) = 0.7 or, $P(\bar{A}) = 0.7$

$\therefore$ 1 – P(A) = 0.7 $\Rightarrow$ P(A) = 0.3

Now, P(B/A) = $\frac{P(A \cap B)}{P(A)}$

$\Rightarrow$ $0.5 = \frac{P(A \cap B)}{0.3} \Rightarrow P(A \cap B) = 0.15$

$\therefore \qquad P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.15}{0.7} = \dfrac{3}{14}$

and, $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.7 - 0.15 = 0.85$

Q.12. Find the general solution of the differential equation $\dfrac{dy}{dx} = e^{x+y}$.

Ans. $\dfrac{dy}{dx} = e^x.e^y$

$\dfrac{dy}{e^y} = e^x dx$

Integrate both sides

$\int e^{-y} dy = \int e^x dx$

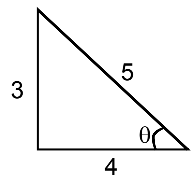
$-e^{-y} = e^x + c$

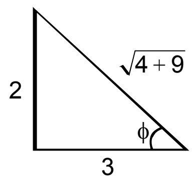
SECTION C

Question numbers 13 to 23 carry 4 marks each.

Q.13. Find the value of sin $\left(\cos^{-1}\dfrac{4}{5} + \tan^{-1}\dfrac{2}{3}\right)$.

Ans. sin $\left(\cos^{-1}\dfrac{4}{5} + \tan^{-1}\dfrac{2}{3}\right)$



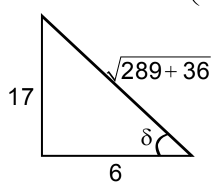


$$\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3} = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\frac{2}{3}$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{9+8}{12}}{\frac{12-6}{12}}\right)$$

$$= \tan^{-1}\left(\frac{17}{6}\right)$$



$$\sin\left(\tan^{-1}\frac{17}{6}\right)$$

$\sin\left(\sin^{-1}\frac{17}{\sqrt{325}}\right)$

$= \frac{17}{\sqrt{325}}$

Q.14. Using properties of determinants, show that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

Ans.

Let $\Delta = \begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \quad \Delta = \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix}$

$\Rightarrow \quad \Delta = (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$

$\Rightarrow \quad \Delta = (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & -b+a \\ 0 & -c+a & 2c+a \end{vmatrix}$ [Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$]

$\Rightarrow \quad \Delta = (a+b+c)\begin{vmatrix} 2b+a & -b+a \\ -c+a & 2c+a \end{vmatrix}$ [Expanding along C_1]

$\Rightarrow \quad \Delta = (a+b+c)\{(2b+a)(2c+a)-(-b+a)(-c+a)\}$

$\Rightarrow \quad \Delta = (a+b+c)\{(4bc+2ab+2ca+a^2)-(bc-ab-ac+a^2)\}$

$\Rightarrow \quad \Delta = (a+b+c)(3bc+3ab+3ca)$

$\Rightarrow \quad \Delta = 3(a+b+c)(ab+bc+ca).$

Q.15. Check whether the relation R defined on the set A = {1,2,3,4,5,6,} as R ={(a,b): b = a + 1} is reflexive, symmetric or transitive.

OR

Let f : N $\to$ Y be a function defined as f(x) = 4x + 3,
where Y = {y $\in$ N : y = 4x + 3, for some x $\in$ N }. Show that is f is invertible. Find its inverse.

Ans. R = {(a,b) : b = a + 1 }

Reflexive –

$(a,a) \in R$

$a \neq a+1$

It is not reflexive relation

Symmetric : -

(a, b) $\in$ R

b = a + 1 …………..(1)

(b, a) $\in$ R

a = b + 1 …………..(2)

by eq. (1) & (2)

(b, a) $\notin R$

It is not symmetric reaction

Transitive : -

(a, b) $\in R$

b = a + 1 ....(3)

(b,c) $\in$ R

c = b + 1 ....(4)

add eq. (3) & (4)

b + c = a + b + 2

c = a + 2

(a,c) $\notin R$

It is not transitive relation

OR

Ans. Consider an arbitrary element y of Y. By the definition of Y, y = 4x + 3, for some x in the domain N. This shows that $x = \frac{(y-3)}{4}$. Define g : Y $\to$ N by $g(y) = \frac{(y-3)}{4}$.

Now, gof(x) = g(f(x)) = g $(4x+3) = \frac{(4x+3-3)}{4} = x$ and

fog(y) = f(g(y)) $= f\left(\frac{(y-3)}{4}\right) = \frac{4(y-3)}{4} + 3 = y - 3 + 3 = y$. This shows that gof = I_N and fog = I_Y, which implies that f is invertible and g is the inverse of f.

Q.16. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, Prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

OR

If$(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

Ans. We have,

$$x\sqrt{1+y} + y\sqrt{1+x} = 0$$

$$\Rightarrow \quad x\sqrt{1+y} = -y\sqrt{1+x}$$

$$\Rightarrow \quad x^2(1+y) = y^2(1+x) \qquad \text{[On squaring both sides]}$$

$$\Rightarrow \quad (x + y)(x - y) = -xy(x - y)$$

$$\Rightarrow \quad x + y = -xy \qquad [\because x \neq y]$$

$$\Rightarrow \quad y(1 + x) = -x$$

$$\Rightarrow \quad y = -\frac{x}{1+x}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left\{\frac{(1+x).1 - x(0+1)}{(1+x)^2}\right\}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{(1+x)^2}$$

OR

Ans. Given $(\cos x)^y = (\sin y)^x$

taking $\log_e$ both side

$\log(\cos x)^y = \log_e(\sin y)^x$

$y\log(\cos x) = x\log_e(\sin y)$

Differentiating both side

$$y.\frac{1}{\cos x}\times(-\sin x)+\log_e(\sin x)\frac{dy}{dx} = x\frac{1}{\sin y}.\cos y.\frac{dy}{dx}+\log_e(\sin y)$$

$$\left(\log_e(\sin x)-x\cot y\right)\frac{dy}{dx} = \log_e \sin y + y\tan x$$

$$\frac{dy}{dx} = \frac{\log_e \sin y + y\tan x}{\log_e \sin x - x\cot y}$$

Q.17. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

and hence evaluate $\int_0^{\pi/2} \frac{x}{\sin x+\cos x}dx$

Ans. Proof : $\int_0^a f(a-x)dx$

Put $a - x = t$

$-dx = dt$

$$-\int_a^0 f(t)dt = \int_0^a f(t)dt$$

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx \qquad \left[\because \int_a^b f(x)dx = \int_a^b f(x)dt\right]$$

Let $I = \int_0^{\pi/2}\frac{x}{\sin x+\cos x}dx$ ....(i)

$$\Rightarrow \quad I = \int_0^{\pi/2}\frac{\frac{\pi}{2}-x}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$\Rightarrow \quad I = \int_0^{\pi/2}\frac{\frac{\pi}{2}-x}{\cos x+\sin x}dx \qquad \text{...........(ii)}$$

Adding (i) and (ii), we get

$$\Rightarrow \quad 2I = \int_0^{\pi/2}\frac{x+\frac{\pi}{2}-x}{\sin x+\cos x}dx = \frac{\pi}{2}\int_0^{\pi/2}\frac{1}{\sin x+\cos x}dx$$

$$\Rightarrow \quad 2I = \frac{\pi}{2}\int_0^{\pi/2}\frac{1}{\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}+\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}dx$$

$$\Rightarrow \quad 2I = \frac{\pi}{2}\int_0^{\pi/2}\frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}dx = \frac{\pi}{2}\int_0^{\pi/2}\frac{\sec^2\frac{x}{2}}{-\tan^2\frac{x}{2}+2\tan\frac{x}{2}+1}dx$$

Let $\tan\frac{x}{2} = t.\text{Then}, d\left(\tan\frac{x}{2}\right) = dt$

$\Rightarrow \quad \frac{1}{2}\sec^2\frac{x}{2}dx = dt \Rightarrow dx = \frac{2dt}{\sec^2\frac{x}{2}}$

Also, x = 0 $\Rightarrow$ t = tan 0 = 0 and x = $\frac{\pi}{2} \Rightarrow t = \tan\frac{\pi}{4} = 1.$

$$\therefore \quad 2I = \frac{\pi}{2}\int_0^1 \frac{2dt}{-t^2+2t+1} = \pi\int_0^1 \frac{dt}{-\left[t^2-2t-1\right]} = \pi\int_0^1 \frac{dt}{-\left[(t-1)^2-2\right]}$$

$$\Rightarrow \quad 2I = \pi\int_0^1 \frac{dt}{\left(\sqrt{2}\right)^2-(t-1)^2} = \pi.\frac{1}{2\sqrt{2}}\left[\log\left|\frac{\sqrt{2}+t-1}{\sqrt{2}-t+1}\right|\right]_0^1$$

$$\Rightarrow \quad 2I = \frac{\pi}{2\sqrt{2}}\left[\log 1 - \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right] = -\frac{\pi}{2\sqrt{2}}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \frac{\pi}{2\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$

$$\Rightarrow \quad 2I = \frac{\pi}{2\sqrt{2}}\log\left\{\frac{\left(\sqrt{2}+1\right)^2}{\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)}\right\} = \frac{\pi}{2\sqrt{2}}\log\left(\sqrt{2}+1\right)^2 = \frac{\pi}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$$

$$\Rightarrow \quad I = \frac{\pi}{2\sqrt{2}}\log\left(\sqrt{2}+1\right).$$

Q.18. Find $\int \frac{x^2+x+1}{(x+2)\left(x^2+1\right)}dx$

Ans. The integrand is a proper rational function. Decompose the rational function into partial fraction. Write

$$\frac{x^2+x+1}{\left(x^2+1\right)\left(x^2+2\right)} = \frac{A}{x+2} + \frac{Bx+C}{\left(x^2+1\right)}$$

Therefore, $x^2 + x + 1 = A(x^2 + 1) + (Bx + C)(x + 2)$

Equating the coefficients of x^2, x and of constant term of both sides, we get A + B = 1, 2B + C = 1 and A + 2C = 1. Solving these equations, we get $A = \frac{3}{5}, B = \frac{2}{5}$ and $C = \frac{1}{5}$

Thus, the integrand is given by

$$\frac{x^2+x+1}{\left(x^2+1\right)(x+2)} = \frac{3}{5(x+2)} + \frac{\frac{2}{5}x+\frac{1}{5}}{x^2+1} = \frac{3}{5(x+2)} + \frac{1}{5}\left(\frac{2x+1}{x^2+1}\right)$$

Therefore, $\int \frac{x^2+x+1}{\left(x^2+1\right)(x+2)}dx = \frac{3}{5}\int\frac{dx}{x+2} + \frac{1}{5}\int\frac{2x}{x^2+1}dx + \frac{1}{5}\int\frac{1}{x^2+1}dx$

$$= \frac{3}{5}\log|x+2| + \frac{1}{5}\log\left|x^2+1\right| + \frac{1}{5}\tan^{-1}x + C$$

Q.19. Solve the differential equation :

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

OR

Solve the differential equation:

$$\frac{dy}{dx} = -\left[\frac{x+y\cos x}{1+\sin x}\right]$$

Ans. We are given that

$\Rightarrow \quad x\dfrac{dy}{dx} = y - x\tan\dfrac{y}{x}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{y}{x} - \tan\left(\dfrac{y}{x}\right)$ ....(i)

Clearly, the given differential equation is homogeneous. Putting y = vx and

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ in (i), we get

$v + x\dfrac{dv}{dx} = v - \tan v$

$\Rightarrow \quad x\dfrac{dv}{dx} = -\tan v$

$\Rightarrow \quad \cot v\,dv = \dfrac{-dx}{x}$, if $x \neq 0$ [By separating the variables]

$\Rightarrow \quad \int \cot v\,dv = \int \dfrac{-dx}{x}$ [Integrating both sides]

log |sin v| = – log |x| + logC

$\Rightarrow \quad |\sin v| = \left|\dfrac{C}{x}\right|$

|sin(y/x)| = |C/x|

Hence, $\left|\sin\dfrac{y}{x}\right| = \left|\dfrac{C}{x}\right|$ gives the required solution.

OR

Ans. $\dfrac{dy}{dx} = -\left(\dfrac{x + y\cos x}{1+\sin x}\right)$

$\dfrac{dy}{dx} = \dfrac{-x}{1+\sin x} - \dfrac{y\cos x}{1+\sin x}$

$\dfrac{dy}{dx} + \dfrac{\cos x}{1+\sin x}y = \dfrac{-x}{1+\sin x}$

Comparing with linear D.E form : $\dfrac{dy}{dx} + py = Q$

$P = \dfrac{\cos x}{1+\sin x}, Q = \dfrac{-x}{1+\sin x}$

Integrating factor, I.F. = e = $e^{\int pdx} = e^{\int \frac{\cos x}{1+\sin x}dx} = e^{\log(1+\sin x)} = 1 + \sin x$

General solution

$y.(\text{ I.F.}).\ \int (Q.(I.F.))\,dx + c$

y.(1 +sinx)= $\int \dfrac{-x}{(1+\sin x)}.(1+\sin x)dx + c$

$y.(1+\sin x) = \int -xdx + c$

$y.(1+\sin x) = \dfrac{-x^2}{2} + c$

Q.20. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

Ans. $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$

Unit vector along sum of $\vec{b}$ & $\vec{c}$ = $\dfrac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|} = \vec{d}$

$$\vec{b}+\vec{c} = (2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\vec{d} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)\hat{i} + 6^2 - 2^2}}$$

A.T.P

$$\vec{a}.\vec{d} = 1$$

$$(\hat{i}+\hat{j}+\hat{k}).\frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 36 + 4}} = 1$$

$$(2+\lambda) + 6 - 2 = \sqrt{(2+\lambda)^2 + 40}$$

$$\lambda + 6 = \sqrt{(2+\lambda)^2 + 40}$$

Squaring both side

$$(\lambda+6)^2 = (\lambda+2)^2 + 40$$

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 4 + 40$$

$$8\lambda = 8$$

$$\lambda = 1$$

Unit vector along $\vec{b}+\vec{c}$ is $\vec{d}$ = $\dfrac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}} = \dfrac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + 2^2}}$ = $\dfrac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$

Q.21. If (a + bx) $e^{y/x} = x$, then prove that

Ans. (a + bx) $e^{y/x} = x$

$$\frac{a}{x} + b = \frac{1}{e^{y/x}} = e^{-y/x}$$

Diff both side w.r.t. x

$$\frac{-a}{x^2} + 0 = -e^{-y/x}\left(\frac{x\frac{dy}{dx} - y}{x^2}\right)$$

$$\left(x\frac{dy}{dx} - y\right)e^{-y/x} = a$$

Diff. both side w.r.t.x

$$-\left(x\frac{dy}{dx} - y\right)e^{-y/x}\frac{\left(x\frac{dy}{dx} - y\right)}{x^2} + e^{-y/x}\left(x\frac{dy^2}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx}\right) = 0$$

$$e^{-y/x}\frac{\left(x\frac{dy}{dx} - y\right)^2}{x^2} = e^{-y/x}\left(x\frac{d^2y}{dx^2}\right)$$

$$\left(x\frac{dy}{dx} - y\right)^2 = x^3\frac{d^2y}{dx^2}$$

Hence Proved

Q.22. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of its edge is 12 cm?

Ans. Let x be the side, V be the volume & S be the surface area of the cube. Then, $V = x^3$, $S = 6x^2$, Where x is a function of time t

$$\frac{dv}{dt} = 8cm^3 / s \ (\text{given})$$

$$8 = \frac{dv}{dt}$$

$$8 = 3x^2.\frac{dx}{dt}$$

So $$\frac{dx}{dt} = \frac{8}{3x^2}$$

Now, $$\frac{ds}{dt} = \frac{d}{dt}\left(6x^2\right) = 12x.\frac{dx}{dt}$$

$$= 12x.\frac{8}{3x^2} = \frac{32}{x}$$

Hence, where x = 12 cm

$$\frac{ds}{dt} = \frac{32}{12} = 2.67 \ cm^2 / s.$$

So surface area is increasing at the rate of 2.67 cm^2/s.

Q.23. Find the cartesian and vector equations of the plane passing through the points A(2,5,–3),B(–2,–3,5) and C(5,3,–3).

Ans. Eq. of plane passing through 3 points

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0 \quad \Rightarrow \quad \begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$

$$16(x-2) - 24(y-5) + 32(z+3) = 0 \Rightarrow 2(x-2) - 3(y-5) + 4(z+3) = 0$$

$$2x - 3y + 4z + 23 = 0$$

SECTION D

Question numbers 24 to 29 carry 6 marks each.

Q.24. Find the point on the curve $y^2 = 4x$, which is nearest to the point (2, – 8).

Ans. Point (x_1, y_1) on curve

$$y_1^2 = 4x_1$$

$$D = \sqrt{\left(x_1-2\right)^2 + \left(y_1+8\right)^2}$$

D is always positive

$$D^2 = \left(x_1-2\right)^2 + + \left(y_1+8\right)^2 = \left(\frac{y_1^2}{4}-2\right)^2 + \left(y_1+8\right)^2$$

$$\frac{dD^2}{dy_1} = 2\left(\frac{y_1^2}{4}-2\right)\frac{2y_1}{4} + 2\left(y_1+8\right)$$

$$= \frac{y_1\left(y_1^2-8\right)}{4} + 2y_1 + 16$$

$$= y_1^3 - 8y_1 + 8y_1 + 64$$

$$= y_1^3 + 64$$

$$\frac{dD^2}{dy_1} = 0 \qquad y_1^3 = -64$$

$$y_1 = -4$$
$$(y_1)^2 = 4x_1$$
$$\Rightarrow\ (-4)^2 = 4x_1 \Rightarrow x_1 = 4$$

Point is (4, – 4)

Q.25. Find $\int_1^3 \left(x^2 + 2 + e^{2x}\right)dx$ as the limit of sums.

OR

Using integration, find the area of the triangular region whose sides have the equation y = 2x + 1, y = 3x + 1 and x = 4.

Ans. $\int_1^3 \left(x^2 + 2 + e^{2x}\right)dx$

by definition

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a) + f(a+h) + \text{........}f(a+(n-1)h)\right]$$

Where, $h = \frac{b-a}{n}$

Here a = 1, b = 3, f(x) = $x^2 + 2 + e^{2x}$ $h = \frac{3-1}{n} = \frac{2}{n}$

$$\int_1^3 \left(x^2 + 2 + e^{2x}\right)dx = 2\lim_{n\to\infty}\frac{1}{n}\left[f(1) + f\left(1+\frac{2}{n}\right) + f\left(1+\frac{2}{n}\right) + \text{......}f\left(1+\frac{2(n-1)}{n}\right)\right]$$

$$= 2\lim_{n\to\infty}\left[\left(1+2+e^2\right) + \left(\left(1+\frac{2}{n}\right)^2 + 2 + e^{2\left(1+\frac{2}{n}\right)}\right) + \text{........} + \left(\left(1+\frac{2(n-1)}{n}\right)^2 + 2 + e^{2\left(1+\frac{2(n-1)}{n}\right)}\right)\right]$$

$$= 2\lim_{n\to\infty}\frac{1}{n}\left[\left(1 + \left(1+\frac{2}{n}\right)^2 + \text{........}\left(1+\frac{2(n-1)}{n}\right)^2\right) + \left(2+2+\text{..........}\ n + \text{times}\right) + \left(e^2 + e^{2+\frac{4}{n}} + \text{......}e^{2+\frac{4(n-1)}{n}}\right)\right]$$

$$= 2\lim_{n\to\infty}\frac{1}{n}\left[1 + 1 + \frac{2^2}{n^2} + 2.\frac{2}{n} + \text{..........}1 + \frac{2^2(n-1)^2}{n^2} + 2.2\frac{(n-1)}{n}\right] + 2n + \left(e^2 + e^2.e^{4/n} + \text{........}e^2.e^{\frac{4(n-1)}{n}}\right)$$

$$= 2\lim_{n\to\infty}\frac{1}{n}\left[1+1+1+\text{..........}n\ \text{times}\right] + \left(\frac{2^2}{n^2} + \frac{4^2}{n^2} + \text{......}\frac{2^2(n-1)^2}{n^2}\right) + 2\left(\frac{2}{n} + \frac{4}{n} + \text{......}\frac{2.(n-1)}{n}\right)$$

$$+2n + e^2\left(1 + e^{\frac{4}{n}} + \text{...........} + e^{\frac{4(n-1)}{n}}\right)$$

$$= 2\lim_{n\to\infty}\frac{1}{n}\left[n + \frac{2^2}{n^2}\left(1 - 2^2 + 3^2 + \text{.....}(n-1)^2\right) + \frac{2.2}{n}\left(1+2+3+\text{......}(n-1)\right) + 2n + e^2\left(\frac{1.\left(1-\left(e^{4/n}\right)^n\right)}{1-e^{4/n}}\right)\right]$$

$$= 2\lim_{n\to\infty}\frac{1}{n}\left[n + \frac{2^2}{n^2}\left(\frac{(n-1)n(2n-1)}{6}\right) + \frac{4}{n}\left(\frac{(n-1)n}{2}\right) + 2n + e^2\left(\frac{\left(1-e^4\right)}{1-e^{4/n}}\right)\right]$$

$$= 2\lim_{n\to\infty}\left[1+\frac{2^2}{n^3}\frac{\left(n\left(1-\frac{1}{n}\right)n.\,n\left(2-\frac{1}{n}\right)\right)}{6}+\frac{4}{n^2}\left(\frac{n\left(1-\frac{1}{n}\right)n}{2}\right)+2+\frac{e^2}{4}\left(\frac{\left(1-e^4\right)}{\frac{1-e^{4/n}}{4/n}}\right)\right]$$

As $n\to\infty, \frac{1}{n}\to 0$

Hence

$$\int_1^3\left(x^2+2+e^{2x}\right)dx = 2\left(1+\frac{4(2)}{6}+2+2+\frac{e^6-e^2}{4}\right)=\frac{38}{3}+\frac{e^6-e^2}{4}\text{sq.unit}$$

OR

Ans. The given lines are

$y = 2x+ 1$ ………………(1)

$y = 3x +1$ ………………(2)

$z = 4$ ………………(3)

Subtracting (1) from Eq. (2) we get $0 = x$

i.e. $x = 0$

Putting $x = 0$ in Eq. (1) $y = 1$

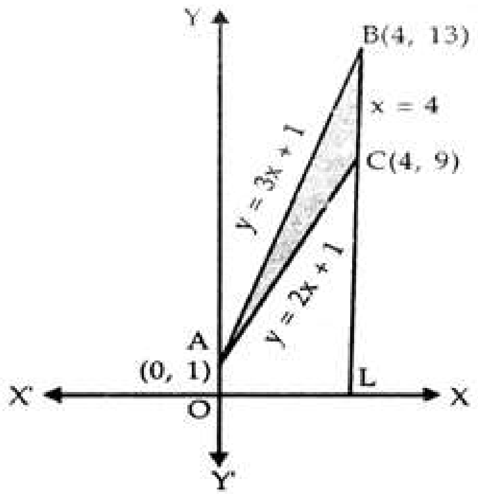
$\therefore$ Lines (2) and (1) intersect at A(0, 1)

Putting $x = 4$ in eq.(2)

$y = 12 + 1 = 13.$

The lines (2) and (3) intersect at B(4, 13)

Putting $x = 4$ in eq. (1)



$y = 8 + 1 = 9$

$\therefore$ Lines (1) and (2) intersect at C(4,9)

Area of Δ ABC = Area of trapezium OLBA – Area of trapezium OLCA ………………(4)

Area of trapezium OLBA = Area of region bounded by OB : $y = 3x + 1$, $x = 0$, $y = 0$ and $x = 4$

$= \int_0^4(3x+1)dx$

$=\left[3\frac{x^2}{2}+x\right]_0^4=\left(\frac{48}{2}-0\right)+4(4-0)$

$= 24 + 4 = 28$ sq. unit

Area of trapezium OLCA = Area of region bounded by OC : $y = 2x + 1$,$x = 0$, $x= 4$, $y = 0$

$=\int_0^4(2x+1)dx=\left[x^2+x\right]_0^4=(16-0)+(4-0)=20$

Putting these values in (4)

Area of ΔABC = 28 – 20 = 8 sq. unit

Q.26. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$, find A^{-1}

Hence solve the system of equations
x + 3y + 4z = 8
2x + y + 2z = 5
and 5x + y + z = 7

OR

Find the inverse of the following matrix, using elementary transformations :

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Ans.

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$$

$$|A| = -1 + 24 - 12 = 11$$

$$A^{-1} = \frac{adj(A)}{|A|}$$

$$adj(A) = \begin{bmatrix} -1 & +8 & -3 \\ +1 & -19 & +14 \\ 2 & +6 & -5 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

The above system of simultaneous linear equations can be written in matrix form as

$$\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$$

or, AX = B

where $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$

Thus, the solution of the system of equations is given by

$$X = A^{-1}B = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -8 & 5 & 14 \\ 64 & -95 & 42 \\ -24 & 74 & -35 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 11 \\ 11 \\ 11 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow$ x = 1, y = 1 and z = 1.

OR

Ans.

$$A = IA$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 5 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 5 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 5 & -2 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_2 \to -R_2, R_1 \to R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -5 & 2 & -1 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$I = A^{-1}.A$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Q.27. A company produces two types of goods, and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g gold. The company can use at the most 9 g of silver and 8 g gold. If each unit to type A brings a profit of ₹ 40 type B ₹ 50, find the number of units of each type that the company should produce to maximize profit. Formulate the above LPP and solve it graphically and also find the maximum profit.

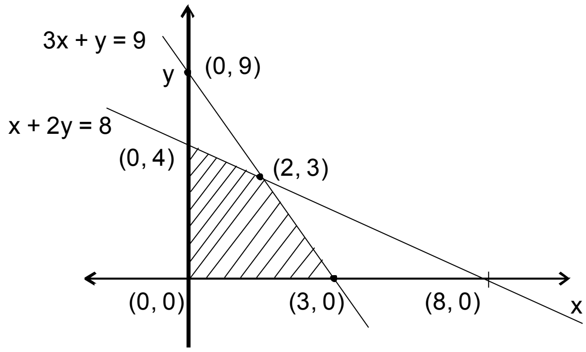
Ans.

	Gold	Silver	Profit
A	1	3	40
B	2	1	50
	8	9	

Let's consider x unit of A and y unit of B

$x + 2y \le 8$ $\quad$ $x, y \ge 0$

$3x + y \le 9$



$x + 2y = 8 \times 3$

$3x + y = 9$

$3x + 6y = 24$

$3x + y = 9$

$- \quad - \quad -$

$5y = 15$

$y = 3$

$x + 6 = 8$

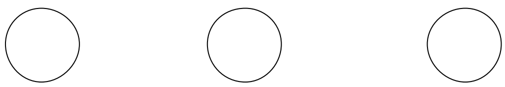
$x = 2$

Point	Z = 40x + 50y
(8,0)	320
(2,3)	80 + 150 = 230
(0,9)	450

Maximum profit if x = 0, y = 9 z = 450

Q.28. There are three coins. One is a two – headed coin, another is a biased coin that comes up heads 75 % of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows heads, what is the probability that it is the two – headed coin?

Ans.



Two - Headed $\qquad$ Baised $\qquad$ Unbiased

Select a coin out of three. $\qquad$ Req. prob. = $\dfrac{\frac{1}{3}\times\frac{100}{100}}{\frac{1}{3}\times\frac{100}{100}+\frac{1}{3}\times\frac{75}{100}+\frac{1}{3}\times\frac{50}{100}}$

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$P(A_1E_1) = 1 = \frac{100}{100}$ $\qquad$ $= \frac{100}{100+75+50} = \frac{100}{225}$

$P(A_1E_1) = \frac{75}{100}$ $\qquad$ $= \frac{4}{9}$

$P(A/E_3) = \frac{1}{2} = \frac{50}{100}$

Q.29. Find the vector and cartesian equations of the plane passing through the points having position vectros $\hat{i}+\hat{j}-2\hat{k}, 2\hat{i}-\hat{j}+\hat{k},$ and $\hat{i}+2\hat{j}+\hat{k}$. and $\hat{i}+2\hat{j}+\hat{k}$. Write the equation of a plane passing through a point (2,3,7) and parallel to the plane obtained above. Hence, find the distance between the two parallel planes.

OR

Find the equation of the line passing through (2, –1, 2) and (5, 3, 4) and of the plane passing through (2, 0, 3), (1, 1, 5) and (3, 2, 4). Also, find their point of intersection.

Ans. Obtained plane is $\vec{r}.(9\hat{i}+3\hat{j}-\hat{k}) = 14$

Now Required plane is parallel to above plane
So, DR. of req. plane are = 9, 3, –1
Hence required plane passing through (2,3,7) & parallel to $\vec{r}.(9\hat{i}+3\hat{j}-\hat{k}) = 14$ is

$9(x-2) + 3(y-3) - 1(y-7) = 0$
$9x - 18 + 3y - 9 - y + 7 = 0$
$9x + 3y - z = 20$

Now two plane are $9x + 3y - z = 14$ ....(1)
$9x + 3y - z = 20$ ....(2)

Distance b/w two parallel plane $= \frac{d_2 - d_1}{\sqrt{a^2+b^2+c^2}} = \frac{20-14}{\sqrt{9^2+3^2+1^2}}$

$= \frac{6}{\sqrt{91}}$ unit

OR

Ans. Eq. of line

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-2}{5-2} = \frac{y+1}{3+1} = \frac{z-2}{4-2}$$

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$$

Eq. of plane

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}$$

$$\begin{vmatrix} x-2 & y-0 & z-3 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$(x-2)(-3) - y(-3) + (-3)(z-3) = 0$
$-3x + 6 + 3y - 3z + 9 = 0$
$-3x + 3y - 3x + 15 = 0$
$-3(x - y + z - 5) = 0$
$x - y + z - 5 = 0$

Intersection point of line and plane is common
$P(3\lambda+2, 4\lambda-1, 2\lambda+2)$

P lies on plane
$3\lambda+2-4\lambda+1+2\lambda+2-5 = 0$
$\lambda = 0$
P(2, –1, 2)
P is intersection point